Error Analysis of Thermal Response Tests (Extended Version)

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1 Introduction

The purpose of a Thermal Response Test (Gehlin 1998, Austin 1998, van Gelder et al. 1999) is to measure the equivalent thermal conductivity of the ground volume tested and thermal resistance of the borehole heat exchanger. The method is based on Fourier's law of heat conduction, which states that the heat flux in a material is proportional to the temperature gradient and thermal conductivity. A borehole heat exchanger of sufficient length with respect to it's radius can be considered as a line source, and the analytical solution of Kelvin's Line Source (Ingersoll & Plass 1948, Carslaw & Jaeger 1959) can be used to solve the heat equation and is widely used to evaluate TRT data. With the line source, by applying a constant heat flux to the ground heat exchanger, the thermal conductivity can be inferred from the constant power rate and the slope of the temperature change with log-time. Once the equivalent thermal conductivity is inferred and far field temperature is measured, the borehole resistance can be derived as well.

The method has been in use as a laboratory technique since at least 1905 (Niven 1905, Stålhane & Pyk 1931) and is well understood. Nevertheless, especially for the field tests, until now a systematic evaluation of the different sources of uncertainty (error) and their effect on the quality of the result has not been made. Some authors have at least characterized the theoretical error of the sensor array (Austin 1998, Witte et al 2002) used for carrying out the test, but other sources of error – such as fluid parameters, heat exchanger length, borehole radius but also model error or standard deviation of the regression coefficients, have so far not been considered.

To estimate the error of a TRT is not so straightforward as it may first seem. First of all, the TRT is based on a model, such as the infinite line source model (ILS), that makes very specific assumptions concerning the process. If any of these assumptions are not true, the measurement procedure cannot be used to obtain estimates of the parameters of interest (equivalent thermal conductivity and borehole resistance). The most important assumption is that conduction of heat is the only heat transport process. For instance, in situations where there is groundwater movement (advection) this is not true and the method cannot be used. Common tests use heat injection at fairly high power rates (> 50 W/m). In these tests thermally induced convection can occur which also invalidates the main assumption of the test. Other assumptions made are that the properties of the medium (thermal conductivity, heat capacity, initial temperature) are isotropic and spatially quasi-constant, that power rate during the test is constant, that the borehole heat exchanger can be represented by a line source and that the internal heat capacity of the borehole heat exchanger can be ignored or that there is no axial heat transport.

Secondly, with a TRT on a single borehole heat exchanger we are not able to obtain a representative sample of the thermal conductivity of the total ground volume, as we only have one single observation of a limited ground volume even if the same borehole is tested more than once. In that sense it is only a crude approximation to treat the result with classical statistical theory as an estimate of the true thermal conductivity of the ground, with an associated standard deviation. In fact, the thermal conductivity of the ground especially will vary as a function of space and time because the ground is not a homogeneous medium but exhibits variations in composition at different spatial scales. Then it becomes a Geo-statistical problem and probabilistic methods need to be employed (Chiles & Delfiner 1999, Bruno et al 2011). Even in one single test this may affect the result: as the temperature gradient progresses through the ground with time the actual ground volume that is tested increases and the equivalent thermal parameters vary according to its evolution. In an extreme case, for instance a test on a steeply inclined geology such as glacial push ridges, this will lead to inconclusive tests as no final estimate of "the" ground thermal conductivity is possible simply because the approximation of a quasi-constant value does not apply.

Thirdly, the test method itself introduces error, this includes errors in the sensors used or error in the power generation for the constant power pulse. Also changes in ambient conditions or even groundwater movement (rainfall, nearby extractions) introduces error.

For the purpose of this paper I consider only the estimate of the thermal conductivity and borehole resistance of one single test on one single borehole heat exchanger. The ground volume around the borehole heat exchanger that is tested is considered to be sufficiently isotropic and spatially constant in composition, so that the equivalent thermal conductivity coincides with the constant value of the parameter. To what extent this singe estimate of the equivalent ground thermal conductivity at one point location is representative of the real (reservoir) ground thermal conductivity, or how repeated tests on the same borehole should be treated, is not the subject of this paper. The error (estimate of the precision) of this single test can therefore be treated by classical statistics.

So far researches have tried to address several issues that may arise with TRT, such as variable heat rate effects or interrupted tests (Beier & Smith 2003, 2005), ground water flow (Signorelli et al 2007), inappropriate model (Bandos et al 2009, Lamarche & Beauchamp, 2007) or effects of heat capacity of the borehole (Bauer et al 2011a, Bauer et al 2011b). Also vertical profiles of thermal conductivities, that may vary between different strata, have been measured using fiber-optics (Fujii et al 2009). However, an analysis of the different possible error sources and their magnitudes has so far not been made. Austin (1998) and also Witte et al. (2002) present a calculation of the sensor array of the TRT, but that calculation does not consider any other error sources.

In a TRT the parameters of interest (thermal conductivity and borehole resistance) are estimated as a function of other variables that are repeatedly measured during the test, measured once before of after the test or estimated independently. The total error, the difference between the real value of the thermal conductivity and the estimated value, is the complex combination of:

1) Measurement error, the error associated with the precision of the sensors used in the equipment and the variations in measurements carried out repeatedly during the experiment (sampling in time). These errors introduce random variations during the test and thereby reduce the precision.

2) Parameter errors, errors in parameters that are measured once and separately (such as borehole length or fluid density) or that are estimated or obtained from other sources (such as borehole diameter, heat capacity of the fluid). This type of error is more serious, as it does not vary during the experiment but introduces bias in the result.

3) Propagation of the individual errors and the method by which they should be combined.

4) Error of the evaluation model used, the final results are obtained by the application of a theoretical relationship. Even if such relationship is evaluated using the true values of all parameters, the result (the estimate of thermal conductivity and borehole resistance) is still only an approximation of the true values.

In this paper I present a characterization of the errors associated with the first three sources listed above, and will give some general remarks about the approximation by the evaluation model.

2 Methods

Although different models are in use to evaluate the TRT results, the most widely used model is the Infinite Line Source Model (ILS). We therefore take the well-known ILS equation as a starting point and explore in a systematic way the different error sources of the variables and parameters of the equation.

In the following I will treat all errors in principle as standard deviations of the parameter. For many of the parameters involved however it is not possible to define the standard deviation. Then at least the range of the error can be estimated, where values near the centre are more likely to occur than values near the end of the error range (in qualitative terms it is a confidence interval). I will call this the *error range*.

First I will present the ILS equation and the specific parameters of interest, their estimators and to which type of error they contribute. Following this the precision and accuracy of the individual parameters will be discussed, with examples based on common sensor technology or common methods to obtain values of second type of parameters. After the individual parameters have been described a formula will be presented which combines the individual errors to an overall error for the estimate of thermal conductivity and borehole resistance. Finally some general remarks will be presented and some guidelines with regard to improving the TRT itself.

The propagation of errors is calculated using general procedures as outlined in Ellison et. al (2000) and Taylor (1997). For equations with independent parameters *U* and *V* and involving only addition / subtraction the error of the final result *X* can be calculated by adding the individual errors in quadrature:

$$\sigma_X = \sqrt{(\sigma_U)^2 + (\sigma_V)^2} \qquad 1$$

For equations involving multiplications or fractions, the errors are given by:

$$\frac{\sigma_X}{X} = \sqrt{(\frac{\sigma_U}{U})^2 + (\frac{\sigma_V}{V})^2} \qquad 2$$

There are some other simplifying rules, but they are not used here.

For equations where the parameters are not independent, or where the equations cannot be expressed as simple sums, products or fractions of the parameters, a numerical procedure is applied where the values of the parameters are varied by a small amount (usually about 1%) and the effect on the final result calculated. The fractional change in the result is a measure of the sensitivity of the parameter of interest to that parameter, and these are multiplied by the estimated error of the parameter and then added in quadrature to obtain the total composite error:

$$\sigma_X = \sqrt{\sigma_U \left(\frac{\partial X}{\partial U}\right)^2 + \sigma_V \left(\frac{\partial X}{\partial V}\right)^2} \qquad 3$$

The contribution of each partial derivative is estimated with a numerical procedure, where U is varied by a small amount and the effect on X calculated:

$$\frac{\partial X}{\partial U} = \frac{\Delta X}{\Delta U} \qquad 4$$

The total error is then calculated by:

$$\partial X = \sqrt{\left(\partial U \frac{\partial X}{\partial U}\right)^2 + \left(\partial V \frac{\partial X}{\partial V}\right)^{2^2}} \qquad 5$$

A spreadsheet with the calculations as presented in this paper is available on our website (http://www.groenholland.com /en/publications/trt_error.zip)

3 Infinite Line Source Equation

The ILS (Ingersoll & Plass 1948, Carslaw & Jaeger 1959) model includes both the conductivity and the borehole resistance. Gehlin (1998) gives a good review of the basic theoretical development of the ILS as applied to thermal response tests.

The basic equation for the time evolution of the average temperature at the borehole wall is:

$$T_f = \frac{Q}{4\pi\lambda H}\left[\ln\left(\frac{4\frac{\lambda}{C}t}{r_0^2}\right) - \gamma\right] + \frac{Q}{H}R_b + T_g \qquad 6$$

With Q the power rate, estimated by:

$$Q = \frac{\sum_{t=1}^{n} q_v(t)\rho c\left(T_{out}(t) - T_{in}(t)\right)}{n} \qquad 7$$

Where:

q_v	:	volume flow circulation medium	m^3/s
ρ	:	density circulation medium	kg/m^3
c	:	heat capacity circulation medium	J/(kgK)
T_{ret}	:	return temperature circulation medium	^oC
T_{in}	:	injection temperature circulation medium	^oC
T_f	:	average temperature of circulation medium	^oC
T_g	:	far field (ground) temperature	^oC
λ	:	ground thermal conductivity	W/mK
H	:	ground loop length	m
R_b	:	borehole resistance	K/(W/m)
y	:	Eulers constant	-
t	:	time	s
r_0	:	borehole radius	m
k	:	coefficient of the regression T_f with ln(t)	K/ln(s)
C	:	the ground thermal capacity	J/(kgK)

From this equation the thermal conductivity is estimated by calculating the slope *k* of the temperature increase with the log-time and inserting this into:

$$\lambda = \frac{Q}{4\pi Hk} \qquad 8$$

Once the thermal conductivity has been estimated (and the ground temperature measured) the borehole resistance can be calculated by (Bruno et al 2011):

$$R_b = \frac{H}{Q}\left(m - T_g\right) - \left[\frac{1}{4\pi\lambda}\left[\ln\left(\frac{4\frac{\lambda}{C}}{r_0^2}\right) - \gamma\right]\right] \qquad 9$$

where:

m : is the intercept of the slope of the regression T_f with ln(t) K

Table 1 gives an overview of all parameters and the associated type of error.

Table 1. Different parameters and estimators in the ILS analysis of TRT results, indicating the type class of the error: 1: measurement error, 2: parameter estimate error, 3: combination error, 4: model error.

Parameter of interest	Estimator	type class of error				note
		1	2	3	4	
λ_{TPT}	$\lambda = \frac{Q}{4\pi Hk}$			x		Estimated equivalent thermal conductivity at one location
Q	$\frac{\sum_{t=1}^{n}\left(q_v(t)\rho c\left(T_{out}(t) - T_{in}(t)\right)\right)}{n}$			x		Power rate, time dependent
ρ			x			density of the circulation medium,
c			x			heat capacity of circulation medium
Tin(t)		x				Fluid injection temperature
Tret(t)		x				Fluid return temperature
$q_v(t)$		x				fluid volume flow rate
H			x			Length of borehole heat exchanger
k					x	Slope of the regression eq. $T_f(t) = m + k.\ln(t)$
m					x	Intercept of the

							regression eq. $T_f(t) = m + k.\ln(t)$
T_f	$\frac{T_{ret}(t)+T_{in}(t)}{2}$	a			x	x	Fluid average temperature, see below
	$T_g + \frac{\lvert T_{ret}(t)-T_g\rvert - \lvert T_{in}(t)-T_g\rvert}{\ln\left(\lvert T_{ret}(t)-T_g\rvert / \lvert T_{in}(t)-T_g\rvert\right)}$	b					
	$T_g + \frac{p\left(\lvert T_{in}(t)-T_g\rvert^{p+1} - \lvert T_{ret}(t)-T_g\rvert^{p+1}\right)}{(1+p)\left(\lvert T_{in}(t)-T_g\rvert^{p} - \lvert T_{ret}(t)-T_g\rvert^{p}\right)}$	c					
C				x			Mean heat capacity of the ground volume
Tg	$\frac{\sum_{d=1}^{n} T(d)}{n}$ where d = depth			x			mean undisturbed ground temperature (assumed constant and isotropic)
Rb	$R_b = \frac{H}{Q}\left(m - T_g\right) + \left[\frac{1}{4\pi\lambda}\left[\ln\left(\frac{4\frac{\lambda}{C}}{r_0^2}\right) - \gamma\right]\right]$				x		Estimated borehole resistance, assuming constant and isotropic conditions
r_o				x			average borehole radius
t			x				Time

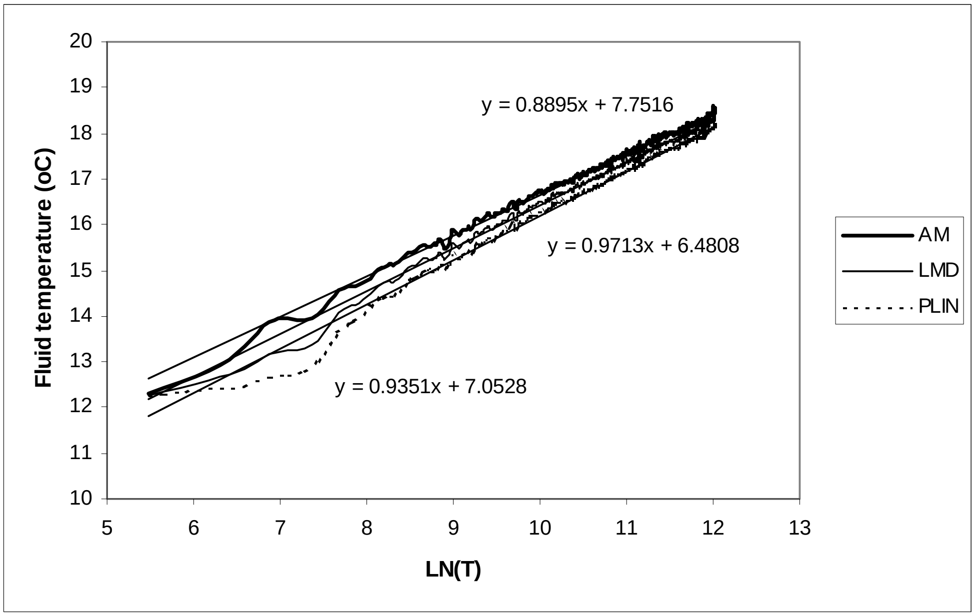
Some remarks about the terms in the table can already be made. First of all, with all parameters one should distinguish between the (unknown) true value *x* and the estimated (measured) value x^*. For the sake of readability and brevity I have not done that.

Secondly, it is worthwhile to note here that the average fluid temperature that is calculated from the fluid injection and fluid return temperatures can be approximated in different ways (Marcotte & Pasquier 2008). The standard method is to calculate the arithmetical mean (a), but this is only correct when the heat flux is constant along the entire borehole, which is not normally a realistic assumption. When a constant temperature on the pipe wall is assumed, the average log mean difference (b) is a good estimator of the steady state average fluid temperature (Incropera & Dewitt 1985).

Marcotte & Pasquier present an equation (c) where they assume the fluid temperature variation at power *p*, $|\Delta T(x)|^p$, varies linearly within the pipe between $|\Delta T_{inject}(x)|^p$ and $|\Delta T_{return}(x)|^p$.

The fact that these different methods to calculate average mean fluid temperatures do not yield equal differences in time (the rate of change is affected) means they will yield different results of estimates of thermal conductivity and borehole resistance as well. An example of the effect of the different averaging methods on the linear regression equation is shown in figure 1. In this typical example, the resulting thermal conductivity values estimated would be: 2,11 (AM average), 1.94 (LMD average) and 2,01 (PLIN average). Also note that, in comparison with the AM method, another parameter (T_g) is introduced that needs to be estimated separately.

Figure 1.Effect of different averaging methods (AM: Arithmetical Mean; LMD: Log Mean Difference; PLIN: P-linear average with p=-0.9).



4 Results

4.1 Measurement errors.

The measurement errors relate to the repeated measurements of the process variables, specifically the measured flow rate, injection and return fluid temperatures. In some cases the electrical power input is measured by a watt-transducer to obtain a direct measurement of power input. The measurement error can be separated in three distinct error types:

1) Accuracy (the closeness of the measured value to the true value). This type of error introduces a bias in the results and should be zero. This is achieved by proper calibration of the sensor system. I assume these errors are zero.
2) Precision, the degree of scatter of the measurement when repeated measurements are made under perfectly constant conditions. This error depends on the characteristic and quality of the sensor system itself and the way in which it is installed in the system.
3) Perturbation of the actual value of the parameter measured, for instance small changes in fluid temperature do occur during measurements. Strictly speaking this is not a measurement error but related to the sampling frequency and how the sensor measures (time-averaging or instantaneous readings).

The measurement error during a test is a result of the precision and perturbation errors. An evaluation of the quality of a test should include a comparison at least of the measured variation with the calculated error range based on the sensor system's precision. Measurements that need to be considered are: flow, injection and return fluid temperature, power input (in the case of watt transducers) and time.

Fluid flow is measured with a flow meter, of which different type exist, with different characteristics. In general volume flow will be measured with an electro-magnetic type flow meter, as this is a robust and easily integrated instrument, other methods include differential pressure, vortex, sonic or mechanical flow meters. It is also possible to directly measure mass flow (by e.g. using a coriolis type flow meter). Errors of flow meters are usually stated as a percentage of flow measured, sometimes with an additional minimum value below a certain threshold. There can be an additional temperature dependence of the error, but this error is very small and ignored here. Table 2 lists some typical errors as given by the manufacturers for different types of flow meters. The absolute error is calculated at a flow rate of 1.5 m^3/hrs and 20 ^oC, this is indication of the maximum error of flow expected in a typical TRT.

Table 2. Relative and absolute errors of different type of flow meters, data from manufacturers.

Sensor type	Relative error %	Absolute error (@ 1.5 m3/hr)
Electro-magnetic (0-1.5m^3/hr) installed in DN50 pipe	±0.33%	± 0.0050 m^3/hr
Coriolis-mass	± 0.15%	± 3.36 kg /hr
Coriolis-volume	± 0.25%	± 0.0004 m^3/hr
Ultrasonic	± 0.50 %	± 0.0075 m3/hr

Fluid injection and return temperature. Temperature can be measured by several different sensors types. Due to its ruggedness, stability of measurement over time and easy of installation PT100 type will normally be used (PT500 or PT1000 are essentially the same but have a different ohmic resistance at 0 ^oC). PT100 sensors are manufactured according to a norm (IEC 60751) and available in different classes, class A tolerance is 0.15 + 0.002|T|; class B tolerance is 0.3 + 0.005|T|.

Form the tolerance statement it is clear that there is a temperature dependence on the precision of the sensor, in the range -50 to +50 ^oC this error is 0.1K, in the range -25 to +25 ^oC it is 0.05K, in the range -5 to +5 ^oC the additional error is 0.01K. Within the typical temperature range of a TRT the total temperature sensor error increases from 0.15 to 0.25K. When calculating the temperature difference the errors can be added in quadrature, the error interval on ΔT ranges from 0.21K to 0.35K.

The error on the temperature measurement is fairly large in view of the most interesting quantity (temperature difference) used for calculating the power rate. It is therefore worthwhile to carefully calibrate the two installed sensors and obtain a matched pair for the temperature difference measurement. In a careful calibration of the actual sensors in the TRT of Groenholland, we achieve a measured error interval on ΔT of ± 0.06K.

Table 3.Typical error of the PT100 temperature sensor in the process temperature range -5 - 50 ^oC.

Sensor type	Relative	Absolute
PT100 @ 0.5 ^oC	± 30.0%	± 0.15K
PT100 @ 50 ^oC	± 0.5%	± 0.25K
PT100 pair, ΔT, @ 20 ^oC, 5K ΔT	± 5.4%	± 0.27K
PT100 matched pair, ΔT, @ 0 ^oC	± 1.2%	± 0.06K

Watt-transducers, with Thermal Response Test utilizing direct electrical heater elements sometimes a watt-transducer is used to measure (electrical) power input and use this as an estimate of thermal power input. Although the precision of these meters can be quite good (relative error range < 2%), not all-electrical power is necessarily completely transferred to the fluid. Also, heat rejection to the fluid by the pump (which is often cooled by the fluid) is not measured. Therefore, the watt transducer is included for completeness but not evaluated further.

Measurement of time. The time drift of data loggers is normally small, especially with regard to the measurement period. Typical clock accuracy for data logger's range between 180s/year and 492 s/year. For a test duration of 100 hours, this would yield a clock error of 8.6 10^{-4} s to 1.6 10^{-3} s. This is so small that it is further ignored.

4.2 Parameter errors.

Included here are parameters that are measured once before the test and parameters that are estimated based on other sources such as literature values. These parameters include the circulation medium density and heat capacity (as well as viscosity and thermal conductivity, but those are not parameters in the ILS equation), borehole heat exchanger length, heat capacity of the ground volume tested and borehole diameter.

Density and heat capacity of the fluid medium are needed in the calculation of the heat rate. The fluid parameters vary with fluid type, mixing ratio and temperature. Due to the dependence on temperature they will vary during the experiment as well. The physical properties of water are well documented, but in Thermal Response Tests other fluids can be used. Especially anti-freeze mixes of water and monoethyleneglycol (MEG) or monopropyleneglycol (MPG) are used. The error in the estimated properties of those mixes then depend on:

1) The physical properties of the pure product, these are obtained from manufacturers data properties (I use data published by DOW chemical) and the accuracy or precision of these data is not known. As the chemical composition of the product is quality-controlled during production one may assume these values to be fairly accurate. Another source of pure-product data are the correlations and mixing rules published by different authors (see Witte, 2010, Haider Kahn 2000 and Melinder 2010 for an overview)
2) The mixing ratio between water and the product. This mixing ratio needs to be estimated. In general the circulation fluid used for a TRT will have a antifreeze content of up to 35% by volume.
3) The variation of the properties with temperature changes during the experiment.

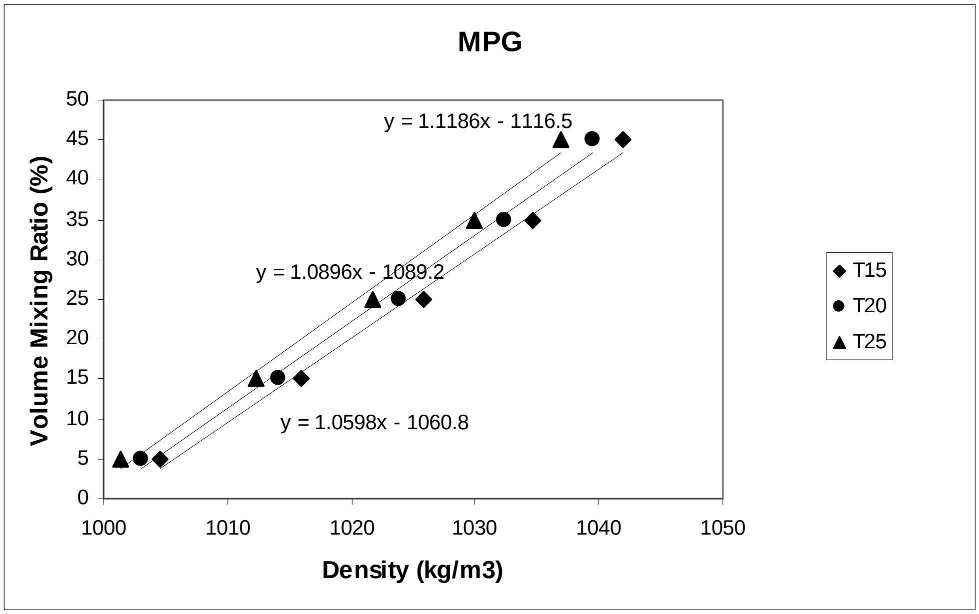
To estimate the mixing ratio a sample from the fluid used in the test is taken and the density and temperature of this sample is measured. With this data the mixing ratio can be estimated from a look-up value in a table of temperature - density data of different mixing ratios. The density can be measured with a precision of about 1:1000 and temperature better than 0.5 ^oC. Figure 2 shows the change in density for MPG and MEG for different mixing ratios at bulk temperatures of 15, 20 and 25 ^oC, table 4 shows the maximum errors of the density and mixing ratio estimates.

Considering the error introduced by the density and temperature measurements the combined maximum error for estimating the mixing ratio for MPG is 1.04% and for MEG 0.98%.

Table 4. Maximum error range in fluid properties (mixing ratio and resulting error in heat capacity) for MPG (35% by volume) and MEG (35% by volume), based on density measurement.

Fluid	Property	Relative	Absolute
MPG	Density	1.5%	16.2 kg/m^3
MPG	Mixing ratio	2.9%	1.0 % point
MPG	Heat capacity	2.0%	90.0 J/kgK
MEG	Density	1.5%	13.8 kg/m^3
MEG	Mixing ratio	2.8%	0.98 % point
MEG	Heat capacity	2.0%	90.0 J/kgK

Figure 2. Relationship between density (kg/m^3) and volume mixing ratio (%) at three different bulk temperatures for MPG (top) and MEG (bottom).



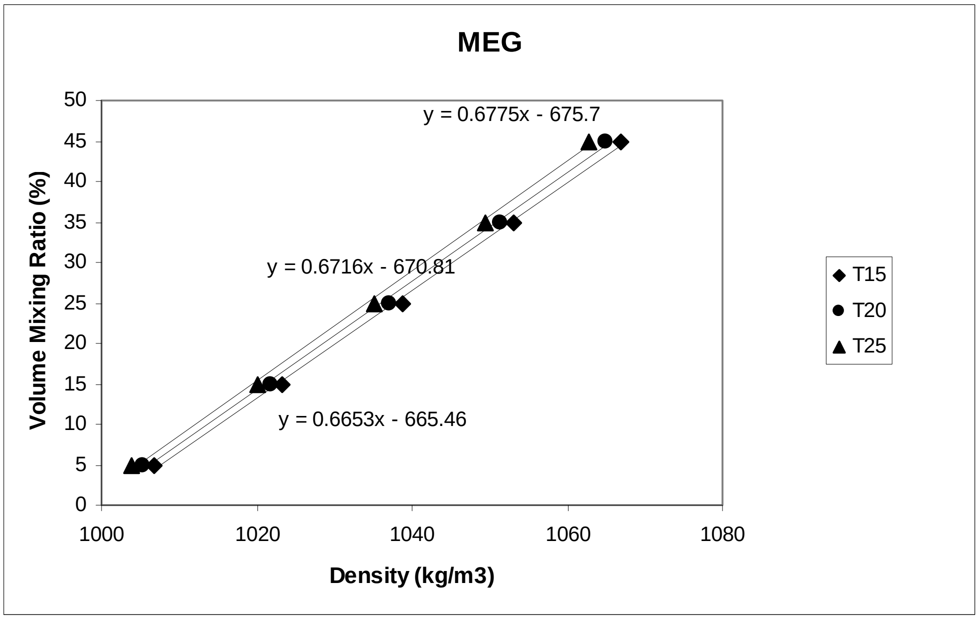
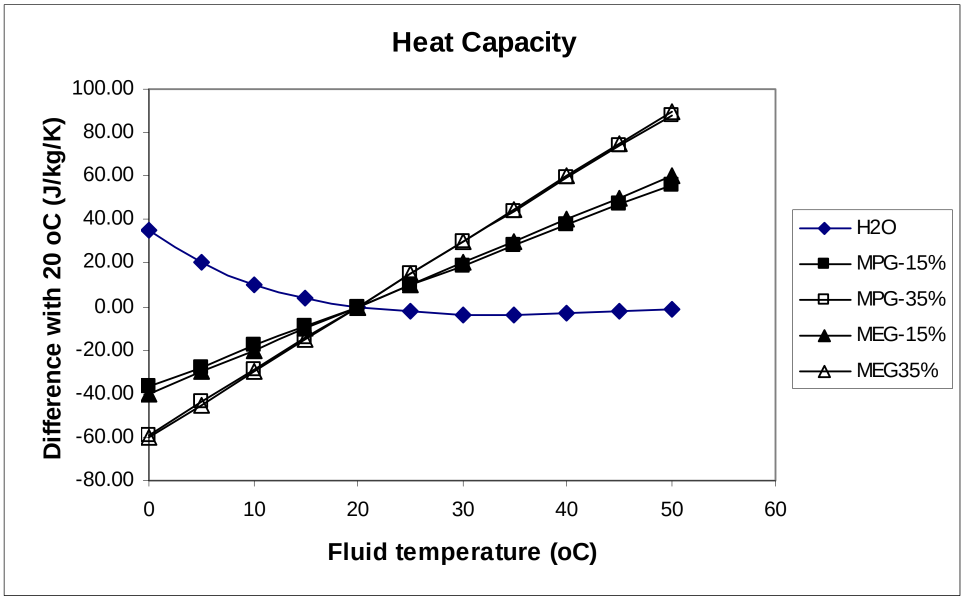
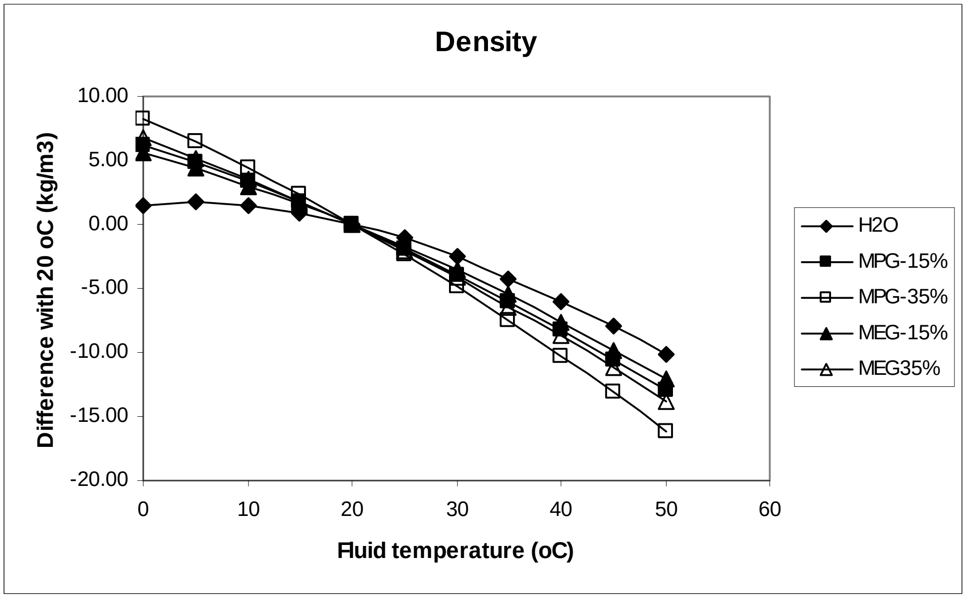


Figure 3. Absolute difference between the fluid properties at different temperatures and taken at 20 °C. Heat capacity (top) and density (bottom).





Once the mixing ratio is known the heat capacity at a specific temperature can be found, assuming the variations in properties of the actual product and the manufacturers data can be ignored, the error in estimated heat capacity as a function of the error in mixing ratio can be calculated. At a bulk temperature of 20 °C the heat capacity of MPG changes at a rate of 12 (J/kgK)/% for MPG and 15.8 (J/kgK)/%, with an error of 1% of mixing ratio this results in an estimated error of heat capacity of 0.3% (MPG) and 0.4% (MEG).

The properties of the fluid are used especially for the calculation of the thermal power, if the fluid properties are taken at a fixed arbitrary constant fluid temperature an additional error will be introduced as the fluid temperature changes during the TRT. To estimate this error we will examine the variation of density and heat capacity with temperature. Figure 3 shows the absolute differences between the fluid properties density and heat capacity at different temperatures compared with the values at 20 ^oC, for water, a 15% and 35% mix of MPG or MEG. The maximum absolute error for the heat capacity is about 90 J/kgK for both MPG and MED at 35% mixing ratio and 50 ^oC bulk temperature (about 2%). Difference in density is -16.2 kg/m^3 (MPG) and -13.8 kg/m^3 (MEG) at 50 ^oC bulk temperature (about 1.5%). A summary of the maximum errors associated with the errors in mixing ratio estimate and variation of parameters with fluid temperature during the test, for density and heat capacity, is given in table 5. Overall error ranges are small and can be minimized by calculating the power rate at every time step using the temperature-corrected fluid properties.

Table 5. Estimated error range in fluid properties when taken at fixed temperature for water, MPG and MEG, values compared with the values at 20 ^oC.

Fluid	Property	Relative	Absolute
Water	Density	1.00%	-10.2
Water	Heat capacity	0.83%	35.0
MPG	Density	1.56%	-16.2
MPG	Heat capacity	2.32%	88.0
MEG	Density	1.31%	-13.8
MEG	Heat capacity	2.51%	90.0

The volumetric heat capacity of the ground (C) is usually not measured but estimated from the geological profile by calculating the weighted average of reference values (with the soil layer thickness as weight). An estimate of the error range in this parameter is not easy to define, but in a range for heat capacity of 2.0 - 3.4 MJ/m^3/K an error range of about ± 0.20 - 0.51 MJ/m^{3}K (a 10-15% error) seems reasonably conservative. A new method (Bruno et al 2011) allows the estimation of the heat capacity together with the borehole resistance. However, the conditional estimation procedure needs limit values and the error range for the heat capacity in the limit range used becomes the error standard deviation.

The active length of the borehole heat exchanger (H). Any TRT should measure the actual active depth of the borehole heat exchanger. With a typical measuring tape a precision of centimeters or even millimeters can be achieved, but it may be accurate only to 20 - 50 centimeters. Calibration of the tape measure should not be forgotten, as the measure used will introduce systematic error in the results (of all tests performed). Moreover, the error will affect results also depending on the length of the loop installed, a 1 meter error on a 20 meter loop will give a much larger error in specific heat rate than the same error on a 100 meter deep loop. Also, when a borehole is not correctly backfilled it may cave-in during the test altering the active length as well as introducing other disturbances.

With the borehole radius (r_o) we need to consider the measurement error as well as the probable variation of the borehole radius over the whole borehole length. In practice the borehole radius will not often be actually measured, but estimated based on drilling rod diameter. Typical borehole radius lies between 0.08 - 0.012 m, with a precision of 0.015 - 0.025 m. It has to be kept in mind however that careless drilling may produce much bigger deviations from the borehole radius (caving).

To calculate the final error we need the **slope coefficient and intercept** of the regression equation. Although these are strictly speaking, according to the classification, model errors, I include a general description of the error now as they will be needed further on.

The error of the regression coefficient cannot be known beforehand as it depends on the time-temperature evolution of the experiment realization. Assuming that the fundamental assumptions of the linear regression hold, the precision of the intercept *m* and slope coefficient *k* can be expressed by their standard deviation. Using the standard deviation of the regression coefficient, the 95% confidence interval can be calculated by: ± 1.96[1] * stdev(*k*).

Typical values for the standard deviation of the regression slope are 0.001 - 0.010 K/ln(s), yielding a 95% confidence interval of 0.002 to 0.020 K/ln(s). The intercept shows typical standard deviations of 0.05 - 0.10 K, yielding confidence intervals of 0.10 - 0.20 K.

4.3 Propagation errors (combination)

In the error calculations I give some examples based on fairly typical values of parameters and individual parameter errors, these values are listed in table 6. In the error equations it is assumed that the individual terms are independent.

Table 6. Reference values for the error ranges of the different measured variables and parameters used for the calculation of the combined errors.

Parameter	**Error range**		**Reference value**
	Absolute	**Reference**	
q_v, volume flow (m^3/hr)	±0.005	0.33%	1.5
ρ, density of medium (kg/m3)	±10.0	1.00%	1000
c, heat capacity of medium (J/(kgK)	±80.0	2.00%	4000
Tin, injection fluid temperature (oC)	±0.15	-	25
Tret, return fluid temperature (oC)	±0.15	-	20
ΔT, temperature difference (K)	±0.212	4.25%	5
Tf, average fluid temperature (oC)	±0.106	*0.53*	-
Tg, far field temperature (oC)	±0.034	0.23%	15
H, loop length (m)	±1.0	1.00%	100
t, time (s)	±4.38 10^{-4}	-	-
r_o, borehole radius (m)	±0.020	26.00%	0.10
C, vol. heat capacity of ground (MJ/(m^3K)	±0.5	20%	2.4
k, slope coefficient	±0.010	1.50%	0.75
m, intercept	±0.100	0.52%	19.5

First some parameters are considered that are made up of either a combination of measurements (temperature difference, average fluid temperature) or are made up of a sequence of measurements (such as average undisturbed ground temperature).

The error in the calculated temperature difference depends on the error in the individual sensors, these are combined:

$$\delta \Delta T_f = \sqrt{(\delta T_{ret})^2 + (\delta T_{in})^2} \qquad 10$$

With a typical sensor error of 0.15K (at 0 °C) this becomes:

$$\delta \Delta T_f = \sqrt{(0.15)^2 + (0.15)^2} = 0.212K \qquad 11$$

At a bulk temperature of 50 °C the error increases to 0.354K.

Here it is assumed that the difference between injection and return temperature is constant (which in a TRT it should be). This may not be always true, for instance during the start of the

[1] *The multiplier is taken from the T distribution and depends on the significance level chosen and the degrees of freedom. As the number of observations (n) in a TRT is large (>> 100) and the degrees of freedom equals n - 2, 1.96 for the 95% and 2.576 for the 99% confidence intervals can be used. Assuming, amongst others, that the errors are distributed normally around the regression line*

heat injection or extraction pulse or due to variations in power. In those cases it may be needed to take into consideration the plug-flow travel time (time lag) and calculate the temperature differences taking into account an appropriate time lag.

Average fluid temperature (T_f). The error standard deviation of the arithmetical mean of fluid temperature is calculated by:

$$\delta T_f = \frac{\sqrt{(\delta T_{ret})^2 + (\delta T_{in})^2}}{2} \qquad 12$$

With a typical sensor error of 0.15K (at 0 ^{o}C) this becomes:

$$\delta T_f = \frac{\sqrt{(0.15)^2 + (0.15)^2}}{2} = 0.106K \qquad 13$$

At a bulk temperature of 50 ^{o}C the error increases to 0.177K.

The error standard deviation of the log mean difference and p-linear average depend also on the undisturbed ground temperature, so I will discuss that first.

In the ideal situation the undisturbed ground temperature is measured by lowering a sensor into the borehole heat exchanger, after this has reached temperature equilibrium with its surroundings, and temperature measurements are taken at regular intervals. Other methods to measure the vertical ground temperature profile exist and may introduce other errors, but these will not be discussed here.

If we only consider the error standard deviation in the measurements and how they add up to the total error in average ground temperature, the estimate of the error standard deviation is:

$$\delta T_g = \frac{\sqrt{\sum_{d=1}^{n} (\delta T_g(d))^2}}{n} \qquad 14$$

Measuring every 5 meters in a 100 meter deep borehole heat exchanger results in an error of 0.034K (using an error of 0.15K for the individual measurements).

To define the errors in the LMD and PLIN averages we need to use the general procedure by taking the partial derivatives as the parameters are not independent. The equations for the combination error standard deviations are:

$$\delta T_f = \sqrt{\left(\frac{\Delta T_f}{\Delta T_g}\delta T_g\right)^2 + \left(\frac{\Delta T_f}{\Delta T_{in}}\delta T_{in}\right)^2 + \left(\frac{\Delta T_f}{\Delta T_{out}}\delta T_{out}\right)^2} \qquad 15$$

The formulas for the error standard deviation of the LMD and PLIN averages are the same, but of course the equation for generating the different solutions (T_f) are not. The final results are:

LMD error

$$\delta T_f = \sqrt{\left(\frac{0.006}{0.150}0.034\right)^2 + \left(\frac{0.101}{0.250}0.150\right)^2 + \left(\frac{0.129}{0.200}0.150\right)^2} = 0.114K$$

PLIN error (with p = -0.9):

$$\delta T_f = \sqrt{\left(\frac{0.011}{-0.150}0.034\right)^2 + \left(\frac{-0.08}{-0.250}0.150\right)^2 + \left(\frac{-0.154}{-0.200}0.150\right)^2} = 0.125K$$

Of course, these errors should be calculated for every time step of an experiment realization and then added again, as the error of the average fluid temperature depends on the Tin and Tout measurements that vary during the experiment. For more precise calculation the dependence of the sensor error on the actual fluid temperature should be taken into account as well.

Now we proceed to the error range of the **thermal power rate Q**. The thermal power rate is calculated by:

$$Q = q_v \rho c \Delta T_f \qquad 16$$

The composite error range on the thermal power rate is given by:

$$\frac{\delta Q}{Q} = \sqrt{\left(\frac{\delta q_v}{q_v}\right)^2 + \left(\frac{\delta \rho}{\rho}\right)^2 + \left(\frac{\delta c}{c}\right)^2 + \left(\frac{\delta \Delta T_f}{\Delta T_f}\right)^2} \qquad 17$$

Using the error range and reference values in table (6), with a reference power rate of 30GJ, we obtain:

$$\delta Q = Q * \sqrt{\left(\frac{0.005}{1.5}\right)^2 + \left(\frac{10}{1000}\right)^2 + \left(\frac{80}{4000}\right)^2 + \left(\frac{0.212}{5}\right)^2} = 30 * 0.048 = 1.44 MJ (400 W)$$

With a heat rate of 30MJ (8.33 kW) the error range is ± 1.44 MJ (400 Watt) or 4.8%. The largest contribution to the error is the measurement of ΔT, effort should be made to achieve as accurate a calibration as possible.

It is important to note that in the power rate there may be another error which is unknown: the pressure loss in the pipe is of course due to the conversion of kinetic energy to friction (heat), as this heat is not measured by the temperature sensors it introduces a bias in the test.

4.4 Error of parameters of interest (combination)

Having defined the measurement errors and errors in other parameters, the error of the final result (estimate of the parameters of interest, thermal conductivity and borehole resistance) depends on how all errors are combined to the final error of the estimate. Error propagation is calculated using the standard rules of combining errors in quadrature. The example calculations use the reference values given in Table 6.

The estimate of **thermal conductivity (λ_{trt})** is obtained by:

$$\lambda_{TRT} = \frac{q_v \rho c \Delta T}{4 \pi H k} \qquad 18$$

And the composite fractional error range can be approximated by:

$$\frac{\delta \lambda_{TRT}}{\lambda_{TRT}} = \sqrt{\left(\frac{\delta q_v}{q_v}\right)^2 + \left(\frac{\delta \rho}{\rho}\right)^2 + \left(\frac{\delta c}{c}\right)^2 + \left(\frac{\delta \Delta T}{\Delta T}\right)^2 + \left(\frac{\delta H}{H}\right)^2 + \left(\frac{\delta k}{k}\right)^2} \qquad 19$$

Using the individual errors and reference values as above, and assuming a value for the thermal conductivity of 2.5, we obtain:

$$\delta\lambda_{TRT} = \lambda_{TRT} * \sqrt{\left(\frac{0.005}{1.5}\right)^2 + \left(\frac{10}{1000}\right)^2 + \left(\frac{80}{4000}\right)^2 + \left(\frac{0.212}{5}\right)^2 + \left(\frac{1}{100}\right)^2 + \left(\frac{0.01}{0.75}\right)^2}$$

= 0.051 * 2.5 = 0.127 W/mK which is about 5.1%.

The largest contribution to the total error (calculated as the contribution to the sum of squares) is the temperature difference (70%) followed by the fluid heat capacity (15.5%) and error on the slope of the regression coefficient (6.9%).

The **borehole resistance (Rb)** is given by:

$$R_b = \frac{H}{Q}\left(m - T_g\right) + \left[\frac{1}{4\pi\lambda}\left[\ln\left(\frac{4\frac{\lambda}{C}}{r_0^2}\right) - \gamma\right]\right] \qquad 20$$

Unfortunately, the definition of the composite error for is not so easy[2]. need to apply a more general procedure and derive the partial derivatives, the uncertainty of the estimate of borehole resistance δRb is then defined as:

$$\partial R_b = \sqrt{\left(\frac{\Delta R_b}{\Delta H}\delta H\right)^2 + \left(\frac{\Delta R_b}{\Delta Q}\delta Q\right)^2 + \left(\frac{\Delta R_b}{\Delta m}\delta m\right)^2 + \left(\frac{\Delta R_b}{\Delta T_g}\delta T_g\right)^2 + \left(\frac{\Delta R_b}{\Delta \lambda}\delta \lambda\right)^2 + \left(\frac{\Delta R_b}{\Delta C}\delta C\right)^2 + \left(\frac{\Delta R_b}{\Delta r_o}\partial r_o\right)^2} \qquad 21$$

Calculating the partial derivatives as before, using a spreadsheet and the typical values of table 6, we obtain:

$$\delta R_b = \sqrt{\left(\frac{-0.0003}{1.2}1\right)^2 + \left(\frac{0.0003}{83.3}400\right)^2 + \left(\frac{0.0019}{0.135}0.15\right)^2 + \left(\frac{-0.0023}{0.158}0.034\right)^2 + \left(\frac{-0.0030}{0.025}0.213\right)^2 + \left(\frac{0.0003}{24000}0.5\right)^2 + \left(\frac{0.00064}{0.001}0.01\right)^2} = 0.027K/(W/m)$$

The total error of Rb in this example is 11.5%. By far the largest contribution to the error is the thermal conductivity, accounting for 93% of the total error. The borehole radius is the second largest (5.8%) followed by the intercept of the regression coefficient (0.66%).

[2] Although the u ncertainty in the first part of the equantion, $\frac{H}{Q}\left(m - T_g\right)$, can be expressed using the simple rules for addition, multiplication and division the second part cannot be expressed as a set of independent functions. For brevity sake I have included the full formula using partial derivatives.

4.5 Model errors

The first consideration is if and to what extent the estimator (3) is a good estimator of the true ground thermal conductivity of the ground volume that is tested (if the true thermal conductivity of the tested ground volume is a good estimator of the reservoir thermal conductivity is another question) and (4) of borehole resistance. This depends on a number of assumptions that are not always possible to test, including (Witte, 2009):

1. The heat transport in the ground is by conduction only
2. The thermal conductivity in the tested ground volume is isotropic and constant in time and space.
3. There is no axial heat transport
4. There is no effect of heat capacity in the borehole
5. The borehole heat exchanger is accurately approximated by a line source
6. There is, after an initial transient state, a steady state borehole resistance
7. The power flux is constant

Some examples of processes that invalidate the above assumptions are: groundwater flow (1), variations in geology and associated thermal conductivities of composite materials, for instance inclusions like clay lenses or gravel beds (2), changing phreatic water table (2, 3, 6), large temperature changes at the surface or due to geothermal gradients (3), large radius boreholes or boreholes filled with high-capacity backfilling (4), short boreholes (5) and fluctuations in power output (7).

Even if all fundamental assumptions hold, there is still a difference between the ILS and the true model. The logarithmic term in the ILS model (1) is only an approximation of the exponential integral. The error is given by (Hëllstrom 1981):

$$\frac{at}{r_0^2} \tag{21}$$

The relative error is < 10% when this value is < 5 and < 2.5% when this value < 20.

The coefficients of the linear regression of slope (k) and intercept (m) are in fact also model errors. The least squares linear regression method that is normally used to obtain estimates of these coefficients also makes definite assumptions about the data, especially: that the relationship is linear, that the errors are normally distributed, uncorrelated and independent, have zero mean and have constant variance. In the case of a TRT there may be nonlinearity introduced by power drift or by changes in ambient conditions (either as a drift or as cyclic effects). Moreover, the errors are not uncorrelated but are auto-correlated in time, therefore the standard deviation of the regression coefficients are not correct estimators of the error. The linear regression model should always be checked for lack of fit and significance of coefficients.

Also the fact that the regression is carried out with log-time, but the sampling takes place at fixed time intervals, introduces a possible error source. The density of observation points will increase as the TRT test time increases, giving relatively more weight to later times. This effect can be mitigated by resampling (or applying appropriate weights to) the data in such a way that the relative data-density does not change. A possible resampling scheme would be to resample the data with constant spacing between observations on the log-time scale (e.g. every 0.15 units) and calculating the required spacing of the sampled data points by taking the inverse of the logarithm. For example, suppose we have 75 hours of data with a sampling frequency of 60 seconds. The logarithmic scale ranges from 4 (first data point) to 12.51. In total there will be 4500 data points, which we can resample on a equidistant log scale by selecting subsequent data points at a distance (in seconds) of e^{lstep} where *lstep* is the value on the log-scale (between 4 and 12.5) with a constant increase yielding 56 equidistant data points. This procedure could be repeated, selecting random starting points, in a bootstrap procedure (Effron and Tibshirani, 1993) to obtain estimates of the standard error of the regression coefficients using all data. Alternatively, e^{lstep} can be used as weights in the regression equation.

The regression should of course still be checked for lack of fit.

The average fluid temperature, especially the way in which this is calculated, is also a model error in the sense that it depends on our assumptions concerning the boundary conditions of fixed temperature or fixed heat flux on the borehole wall. The ILS method of TRT really assumes constant heat flux, but that is probably not realistic. Marcotte & Pasquier (2008) show that a P-linear estimator with p -> -1 gives the best unbiased estimate of average fluid temperature.

5 Conclusions

TRT results are widely used to assess the potential for geothermal systems and to design these systems. Feasibility, cost and performance of the geothermal installations using borehole heat exchangers depends to a large extent on these parameters.

The TRT is in itself a straightforward method, albeit not easy to execute with sufficient accuracy under field conditions. Lacking in current TRT reporting is an evaluation of fundamental assumptions and error evaluation. To be able to successfully apply a TRT result in a project, a TRT report needs to include a chapter on quality control. This chapter needs to give the following information:

- Qualitative assessment of test location and test results with regard to fundamental assumptions of the TRT.
- Estimate of thermal conductivity and borehole resistance based on site geology, these can be used to select appropriate test conditions.
- Calculation (using the TRT machine characteristics and site test conditions) of the theoretical error and observed error. Explanation of any differences between these.
- Explicit choice of formula for calculation of average temperature.
- Examination of regression with regard to lack of fit and error, error of coefficients calculated with bootstrap method where resampling takes into account differences in data-densities.
- Plotting CUSUM (Cumulative SUM) charts of estimated thermal conductivity especially noting if estimates converge to a stable value.

In this paper I have given an overview of the error sources of a Thermal Response Test and have given some example calculations for typical situations. Results show a clear ranking of the magnitudes of the different individual errors in the TRT analyses. Large relative errors are found for the borehole radius (26%), soil heat capacity (20%) and measured temperature difference (4.25%). For the composite errors, for the power rate, especially the temperature difference is important. The error of the thermal conductivity estimate also depends to a large extent on the temperature difference (70%), the fluid heat capacity (15.5%) and the slope error (6.9%).

For the estimate of the borehole resistance the estimated thermal conductivity contributes over 90% to the total error, the borehole radius 5.8% and the intercept of the regression 0.66%.

Note that the error calculations are based on the estimated errors of the different parameters, if there is an issue with the accuracy the result can be quite different. For instance, the estimated undisturbed ground temperature has a small effect on the error of the borehole resistance based on the error of the individual temperature measurements. If this parameter is not measured accurately however, the contribution to the bias of the borehole resistance can be quite large.

The results also indicate a number of possibilities and areas where the error in the TRT can be decreased. A careful calibration of the temperature sensors used to calculate the temperature difference is of main importance. One of the methods to decrease error and bias in the regression line calculation is by resampling the data to obtain an even distribution of observations on the log-time scale. Also the correct choice of method to obtain the average fluid temperature is essential.

Clearly, the experimenter's choice with regard to experiment settings is important. Sometimes selecting a high flow rate is advocated, but this will affect the experiment in two ways. First of all it will decrease the temperature difference, which results in a larger relative measurement error. Moreover, the conversion of pump kinetic to thermal energy (pressure loss), which cannot be measured by the temperature sensors, will also be larger. It is therefore better to select a lower flow rate and higher temperature difference for the experiment.

Further work is needed to incorporate this analysis in a wider scope aimed at understanding the relation between a single test and repeated tests at the same location or interpreting tests performed at several locations. A more detailed and quantitative quality control protocol would need to be developed to allow tests of different test performers to be compared.

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References

Austin, III, W.A. 1998: Development of an in situ system for measurement for ground thermal properties. MSc Thesis, Oklahoma State University. 164 pp.

Bandos, T.V., Montero, Á., Fernándeza, E., Santandera J.L.G, Isidroa, J.M., Péreza, J., Fernández de Córdoba, P.J., Urchueguía, J.F.. 2009 Finite line-source model for borehole heat exchangers: effect of vertical temperature variations. Geothermics, 38: 263-270.

Bauer, D., Heidemann, W., Müller-Steinhagen, H. and Diersch, H.-J. G., 2011a. Thermal resistance and capacity models for borehole heat exchangers. International Journal of Energy Research, 35: 312–320.

Bauer, D., Heidemann, W., Diersch, H.-J.G.. 2011b. Transient 3D analysis of borehole heat exchanger modeling, Geothermics, Volume 40, Issue 4, pp 250-260.

Beier, R.A. & Smith, M.D., 2003. Removing variable heat rate effects from borehole tests. ASHRAE Transactions Vol. 109, Part 2, pp. 463-474

Beier, R.A. & Smith, M.D., 2005. Interrupted In-Situ Tests on Vertical Boreholes. ASHRAE Transactions Vol. 111, Part 1, pp. 702-713.

Bruno, R, Focaccia, S. & Tinti, F., 2011. Geostatistical modeling of a shallow geothermal reservoir for air conditioning of buildings. Proceeding of IAMG 2011 Salzburg, "Mathematical Geosciences at the Crossroads of Theory and Practice, pp. 146-163.

Carslaw, H.S., Jaeger, J.C., 1959. Conduction of heat in solids, second edition. Clarendon Press, Oxford.

Chiles, J.P. & Delfiner, P. 1999. Geostatistics: Modeling Spatial Uncertainty. Whiley, New York. 696 pp.

Effron , B. and Tibshirani, R.J. 1993. An Introduction to the Bootstrap. Chapman & Hall. 436 pp

Fujii, H., Okubo, H., Nishi, K., Itoi, R. Ohyama, K. and Shibata, K., 2009. An improved thermal response test for U-tube ground heat exchanger based on optical fiber thermometers,Geothermics,Vol.38,No.4,pp.399-406,.12.

Gehlin, S. 1998: Thermal Response Test, In-Situ Measurements of Thermal Properties in Hard Rock. Licentiate Thesis, LULEÅ University of Technology. 42 pp.

Haider Kahn, M. 2000. Modeling, simulation and optimization of ground source heat pump systems. BSc Thesis, Oklahoma State University. 218 pp.

Incropera, F.P. & Dewitt, D.P., 1985. Fundamentals of heat and mass transfer, 2nd edition. Wiley New York, 802 pp.

Ingersoll, L.R. and Plass, H.J. 1948: Theory of the Ground Pipe heat Source for the Heat Pump. Heating, Piping & Air Conditioning. July. Pp. 119 – 122.

Lamarche, L. & Beauchamp B., 2007. A new contribution to the finite line-source model for geothermal boreholes. Energy and Buildings, 39(2): 188-198.

Marcotte, D. & Pasquier, P. 2008. On the estimation of thermal resistance in borehole thermal conductivity test. Renewable Energy, 33(11): 2407 – 2415.

Melinder, Å (ed), 2010. Properties of secondary working fluids for indirect systems. International Insititute of Refrigeration (IIR).

Mogeson, P.1983: Fluid to Duct Wall Heat Transfer in Duct System Heat Storages. Proceedings of the International Conference on Subsurface Heat Storage in Theory and Practice. Swedish Council for Building Research.

Niven, C, 1905: On a method of finding the conductivity for heat. Proc. R. Soc. Lond. A. 76: 34-48.

Signorelli, S., Bassetti, S., Pahud, D. & Kohl, T., 2007. Numerical evaluation of thermal response tests. Geothermics, 36(2): 141-166.

Stålhane, B. & Pyk, S., 1931: Ny metod för bestämning av värmeledningskoefficienter. Teknisk Tidskrift, 28, Svenska Teknologföreningen.

Taylor, J.R., 1997 An introduction to error analysis: the study of uncertainties in physical measurements. University Science Books, 327 pp.

Van Gelder, A.J., Witte, H.J.L., Kalma, S., Snijders, A. and R.G.A. Wennekes 1999: In-situ Messungen der thermische Eigenschaften des Untergrunds durch Wärmeentzug. IN: T. Hitziger (Ed): OPET Seminar "Erdgekoppelte Wärmepumpen zum heizen und Klimatisieren von Gebäuden. 109 pp.

Witte, H.J.L. 2006. Geothermal Response Tests using controlled multi-power level heating and cooling pulses (MPL-HCP): Quantifying ground water effects on heat transport around a borehole heat exchanger. In: Stiles (ed). The Tenth International Conference on Thermal Energy Storage, Ecostock 2006 Proceedings. May 31 - June 2, 2006, Stockton College New Jersey (USA).

Witte, H.J.L. 2010. Fluid properties correlations and mixing rules (water, monopropylene glycol and ethylene glycol). Groenholland, 2010. Unpublished report.

Witte, H.J.L., A.J. van Gelder, J.D. Spitler. 2002. In Situ Measurement of Ground Thermal Conductivity: The Dutch Perspective. ASHRAE Transactions, Vol. 108., No. 1.